

Math Practice At Home: A Factor That Positively Influences the Ability to Solve Non-Routine Mathematical Problems?

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Abstract: - *The difficulty in solving math problems across all grades not only is well-known in Greek schools, but also to all over the world. This empirical quantitative research will attempt to check whether practice routine math problems plays a positive role in effectively solving these problems. Twenty-five students from ninth-grade (students 14–15 years old) of a public school asked to solve one non-routine problem similar to PISA's math problems. Then, after collecting the tests, the students who solved it in an acceptable manner identified as strong solvers, while those who did not solve it were identified as weak solvers. Questionnaires were then administered in order to find out the hours that the students spend solving math problems at home while their grades in Mathematics were filled out by the teachers. Using the SPSS statistical package, appropriate statistical measurements were conducted that showed practice is an important factor in effectively solving routine problems.*

Keywords: *Practice, mathematics, non-routine math problems*

1. Introduction

Mathematical problems tasks exist in school mathematics textbooks of all countries in the world. However, the school orientation toward the memorizing skills has resulted in a generation that has had a poor performance in mathematical thinking and problem solving.

Non-routine math problems are exercises that are primarily designed to develop a very specific technique for performing abstract cognition brain processes. Polya (1957) considers problem solving to be a fundamental human ability since most of our conscious thinking is about problems.

The steps of solving a non-routine problem according to Polya's model are, 1st: Understand the problem, is considered a prerequisite, 2nd: Designing a plan toward the solution, namely choosing the strategical approach, 3rd: Implementing the solution plan and 4th: The fourth step is the overview of the solution, where it is checked whether the solution fits the problem data (Johanssen, 2003). Therefore, in order to find the best solution to a problem requires a thorough understanding of its written language form.

The second step, in properly solving a non-routine problem, is choosing the most forthrightly strategy, that is, create data connection to unknowns.

According to various studies, the wrong choice of strategy is mainly due to the small amount of time spent practicing and bring about moderate strategies and not the students' intellectual power. What emerges from research is that improving numerical skills requires more practice and guidance on the use of different strategies (Thom & Pirie, 2002; Lerch, 2004; Nicolaou & Philippou, 2007).

The Problem-Based Learning (PBL) method is a shift on how students learn, not focusing on teacher's instruction but focusing on student's improvement of knowledge acquisition and reasoning skills.

Students learn how to confront non-routine math problems when teachers help them in problem solving processes. That way the students are given the opportunities to investigate personal learning objectives and review their solving procedure results. Consequently, students become good critical thinkers (Hmelo-Silver, 2004).

In addition, the goals of PBL learning environments include facilitating students to develop effective metacognitive strategies, such as control their learning, monitoring their progress and finally evaluate their progress. In other words, to improve the ability to build on the prior knowledge to design a plan for an effective strategy.

Metacognitive strategies are as important as problem solving. They help develop self-directed and long-lasting learning skills, and without their implementation the student will almost certainly not

reach an acceptable solution (Gurat & Medula, 2016).

If teachers want the students to become successful problem solvers, firstly they have to teach them how and then to give them more practice opportunities (Grouws & Cebulla, 2002).

Interesting is the results of Greek students in the PISA (Program for International Student Assessment) mathematics tests showing a lower performance than the OECD average from 2003 up to 2015 (OECD, 2015). This is shown in the Figure 1, below.

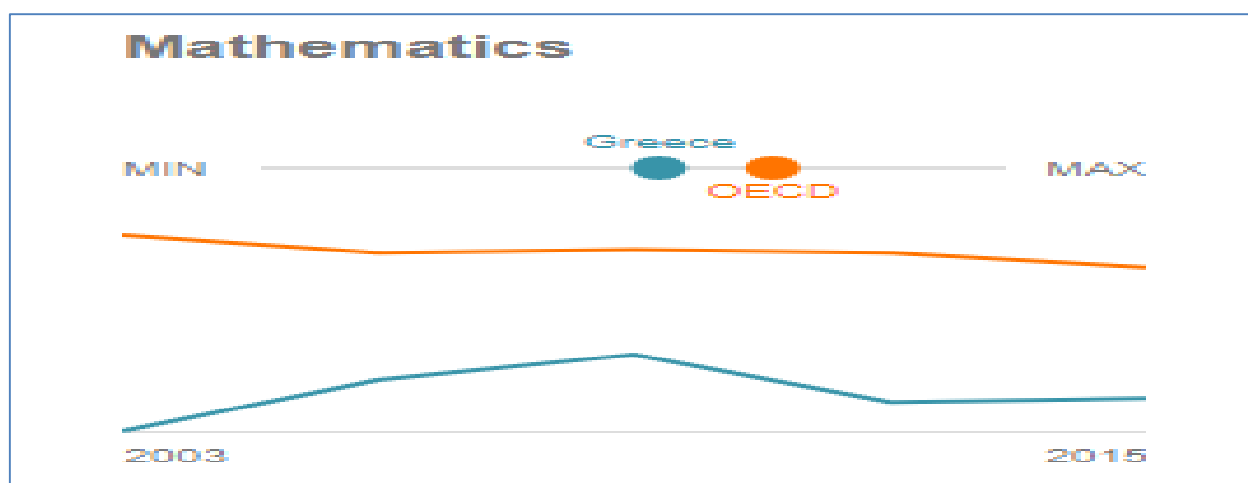


Figure 1. Greek students' math performance compared to OECD countries average.

(Source: <http://www.compareyourcountry.org/pisa/country/GRC?lg=en>)

In addition, the results of the 2018 PISA math test show that “on average, 15-year-olds score 451 points in mathematics compared to an average of 489 points in OECD countries” (OECD, 2018).

The purpose of the present study was to examine whether practicing mathematics non-routine problems helps students develop their solving ability to these types of problems. The triggering event for the research was the poor results of Greek students in mathematics on PISA test.

The results of the research showed that the more students practice mathematical problems, the better their performance in solving them. Teachers should therefore pay more attention to the practice of students with mathematical problems.

2. Methodology

The research is empirical and utilizes the methodological approach of classroom intervention. Twenty-five ninth-grade students participated in the research by random sampling. The non-routine problem that was given to students to solve was similar to PISA exam templates. The purpose of the test was to investigate whether students understand the verbal problems and be able to convey them into mathematics terms.

Students took the test during regular school hours on a typical school day. Just before starting the test, the researchers emphasized the purpose of the research was to study the difficulties students face in solving non-routine problems and that was not an assessment. For students' anonymity, they asked not to write their name on the test or include any other identification but a number in ascending order. After

collecting each test, the teacher wrote the student's grade in mathematics. The researchers engaged SPSS 21 statistical package for data analysis.

Research examined whether the time devoted by the student to practice, in hours per week, related to the effectiveness of the non-routine problem solving. Students divided into "strong" solvers if they solved the problem effectively and "weak" solvers if they did not reach the solution.

In addition, researchers examined the relationship between practice time and their grade in mathematics.

The variable "Grade in maths" contained numbers from 10 to 19, the variable "Training" included numbers from 1 to 10 (in hours), with the following grouping: 1-4 hours "Slightly Satisfactory", 4-7 hours "Satisfactory", and 8-10 hours "Very Satisfactory" and finally the variable "Solving Strategy" had two values, "weak (for the weak solver) and "strong" (for the strong solver).

The problem was this: "Divide the number 12 into two parts such that twice of one part is three points smaller than the other part."

The student who wrote the following answers was characterized as a "strong" solver:

- A. $x + 2x + 3 = 12$ or
- B. If x is the smallest and y the largest then $2x = y - 3 \Rightarrow x + 2x + 3 = 12$, and
- C. The correct numbers were given without writing the equation.

Table 1

95% confidence interval for pairwise mean comparisons on "Training hours"

Training hours	M	SD	Satisfactory	Very Satisfactory
Slightly Satisfactory	12,00	1,240		
Satisfactory	15,69	1,601	-5,01 to -2,37	
Very Satisfactory	17,60	1,140	-7,39 to -3,81	-3,71 to -,10

Next, for the variable "Solving Strategy", it appeared the data didn't follow normal distribution (Shapiro-Wilk test for the value "weak", $p = 0.63 > 0.05$, and

We characterized "weak" solver the student A) if did not write anything B) if the answer was "x is the smallest part then the largest would be $2x - 3$ " and C) any other answer.

3. SPSS Results & Analysis

After examining the data for normality using the Shapiro-Wilk test, indicated normal distribution. One-way ANOVA was used to test how the math test score differentiates depending on the hours of practice at home. The independent variable was "Training Hours" and the dependent variable was the "Grade in maths" for the score on the test. The effect of "Training Hours" (practice hours) on the "Grade in mathematics" (score of the math test) was significant, $F_{2, 29} = 39, 78, p = .00$. The correlation of test scores in mathematics with practice hours, as found by η^2 was strong and the level of practice hours contributed to 73.3% of the variance of the dependent variable.

A significant effect found and because the independent variable has more than 2 values (here, 3, 'Slightly Satisfactory', 'Satisfactory', 'Very Satisfactory'), a post hoc analysis performed.

The Post hoc analysis examined in which pair of group's detected difference between the means. Because the test for homogeneity of variance was $p = 0.69$, the assumption of homogeneity of variance holds and the Bonferroni method used.

There was a significant difference between the mean of the math test scores and the values of the variable "Training hours". The previous results appear below in Table 1.

for "strong", $p = 0.49 > 0.05$). For this reason, researchers used the non-parametric test Mann-Whitney (U).

The following Table 2, shows that the Mann-Whitney (U) tests rejected null hypothesis.

Table 2

Results of the Mann-Whitney U test for the variable “Solving Strategy”

Null Hypothesis	Test	Sig.	Decision
The distribution of “Training hours” is the same across categories of “Solving Strategy”	Mann- Whitney	0,007 ¹	Reject the null Hypothesis
The distribution of “Grade in maths” is the same across categories of “Solving Strategy”	Mann- Whitney	0,001 ¹	Reject the null Hypothesis

¹Exact significances are displayed. The significance level is .05

About the variables “Training hours” and “Solving Strategy”, as shown in Table 2, because the p value in the Mann-Whitney test refers to a two-tailed p value, the result for the one-tailed value p is defined as $0.007 / 2 = 0.0035$. Thus, the null hypothesis (Ho) is rejected ($U = 12.5, p = 0.0035 < 0.05$). Therefore, it is more likely students who practice more in solving non-routine mathematical problems to be strong problem-solvers.

In the next Mann-Whitney test (Table 2), concerning the variables “Grade in maths” and “Solving

Strategy”, the resulting p-value in the Mann-Whitney U test refers to a two-tailed p-value, the one-tailed p-value is set to be $0.001 / 2 = 0.0005$. Based on the previous results the null hypothesis (Ho) is rejected ($U = 10.5, p = 0.0005 < 0.05$). What it means, is more likely students who acquire good scores in math tests to be strong problem-solvers.

The following Figure 2, shows the relation of “weak” and “strong” solvers with the score of the test in mathematics. As we see, “strong” solvers have higher score in the test than the “weak” solvers.

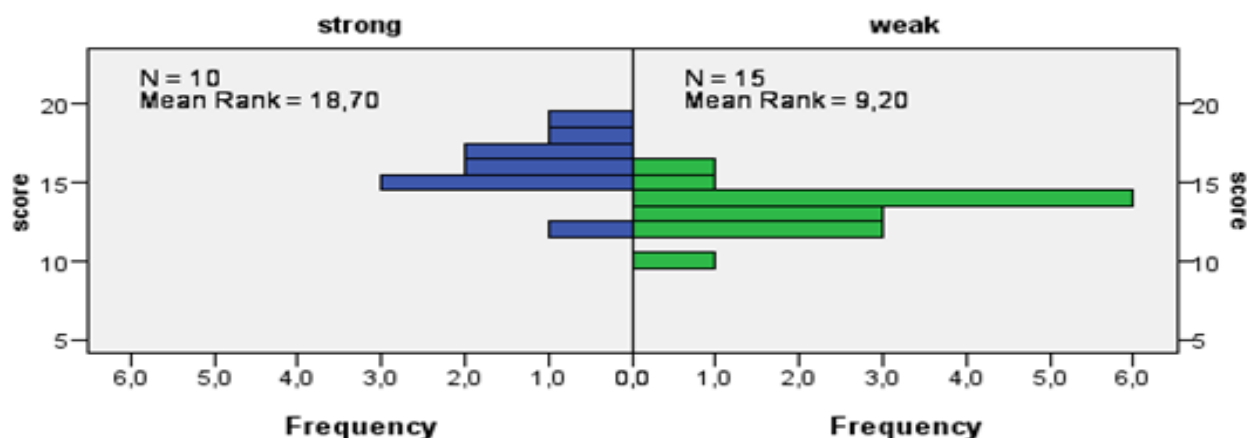


Figure 2. Comparison of Strategic with score in math test

Continuing, the following Figure 3 shows the relationship between the “weak” and the “strong”

solvers with the practice time they spend. As the Figure 3 indicates, “strong” solvers practice more.

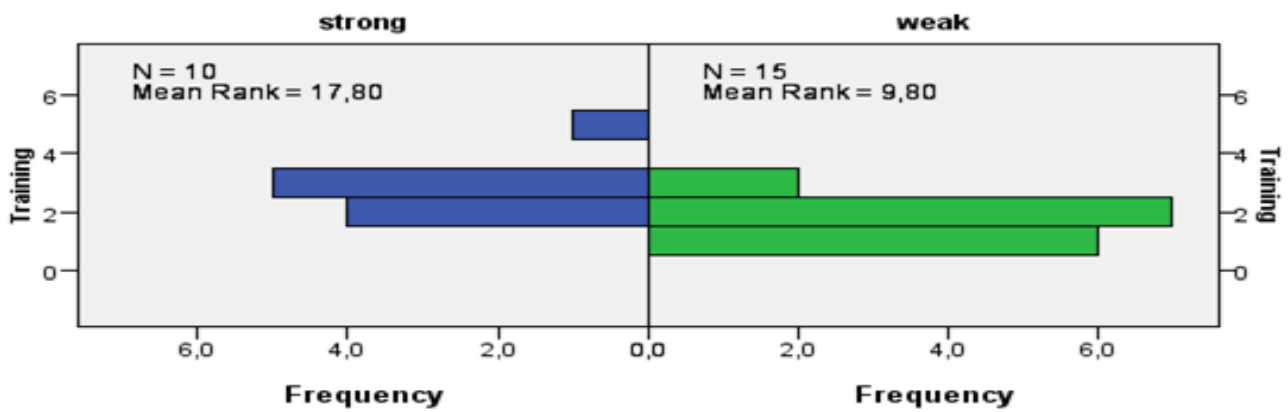


Figure 3. Comparison of strategic with hours of practice

Subsequently, the Spearman test showed a positive correlation of solving strategy with practice and test score. The results depicted in the following Table 3,

makes clear the extent to which the three variables are connected.

		Solving strategy	Training hours	Grade in math's	
Spearman	Solving strategy	Correlation Coefficient	1,000	,579**	,653**
		Sig. (2-tailed)	.	,002	,000
		N	25	25	25
	Training hours	Correlation Coefficient	,579**	1,000	,178
		Sig. (2-tailed)	,002	.	,393
		N	25	25	25
	Grade in maths	Correlation Coefficient	,653**	,178	1,000
		Sig. (2-tailed)	,000	,393	.
		N	25	25	25

** . Correlation is significant at the 0.05 level (2-tailed).

Table 3, shows the correlation coefficients among “Solving strategy”, “Training hours” and “Grade in maths”. The results of the correlation analysis showed that the correlations, “Solving strategy”- “Training hours” and “Solving strategy- “Grade in maths” are both statistically significant and fairly positive (“Training hours” (r (25) = 0.58, p <0.01) and “Grade in maths” (r (25) = 0.65, p <0.01), while positive relationship among “Training hours” and “Grade in maths” exists but it is weak (r (25) = 0.18, p <0.01).

The positive sign shows a positive correlation, that is, the values of one variable increase so do the values of the other variable. This means that students who increase their practice hours are more likely to

become “strong” solvers and increase their math scores.

4. Conclusions & Discussion

The general finding suggests that the students in the research had a great deal of difficulty in the non-routine problems. Specifically, confusion or ignorance of concepts and symbols was observed. There was a difficulty in conceptual understanding of the statements expressing relationships between quantitative variables. For example, there were responses like "twice the number x is x²" or "3 - y, the decrease by three".

Problem solving in terms of equations is a unit taught in all school classes from sixth-grade to ninth-grade. Students in seventh-grade and eighth-grade are

taught for solving problems using first-degree equations. In addition, at grade nine, students are also taught to solve second-degree polynomial equation problems, even study and apply systems of linear equations.

Therefore, the students attending ninth-grade have tackle similar problems several times in the past.

Low performance in non-routine problem demonstrates students' difficulties in understanding and applying mathematical concepts.

In conclusion, the method of teaching math problems in the Greek schools does not contribute to a meaningful understanding of mathematical concepts by students.

However, it appears from the results that when practice solving non-routine problem is more likely to enhance strategies for solving such problems.

Of course, the sample size was limited in order to reach general conclusions. However, the results of the research are consistent with the literature and are an indication of what teachers should apply.

In conclusion, more research is needed to be carried out on the effects of other variables concerning the students' difficulty in problem solving, in general.

References

1. Grouws, D. & Cebulla, K. (2002). *Improving student achievement in mathematics*. (Educational Practices 4). Retrieved from: <http://www.ibe.unesco.org/en/document/improving-student-achievement->
2. Hmello-Silver, C. (2004). Problem-based learning: what and how do students Learn? *Educational Psychology Review*, 16(3), 235-266.
3. Johanssen, D. H. (2003). Designing research based-instructions for story problem. *Educational Psychology Review*, 15(3), 267-296.
4. Polya, G. (1957). *How to solve it: A New aspect of mathematical method*. 2nd Ed. New York: Double Day and Co.
5. Thom, J. S. & Pirie, S. E. B. (2002). Problems, Perseverance, and mathematical Residue. *Educational Studies in Mathematics*, 50, 1–28.
6. Lerch, C. M., (2004). Control decisions and personal beliefs: their effect on solving mathematical problems. *Journal of Mathematical Behaviour*, 23, 21–36.
7. Gurat, M., & Medula C. Jr. (2016). Metacognitive Strategy Knowledge Use through Mathematical Problem Solving amongst Pre-service Teachers. *American Journal of Educational Research*, 4 (2), 170-189. DOI:10.12691/education-4-2-5
8. Nicolaou, A. A. & Philippou, G. N. (2007). Efficacy Beliefs, Problem Posing, and Mathematics Achievement. *Focus on Learning Problems in Mathematics*, 29(4), 48- 70.
9. OECD (2015). OECD Education at a Glance 2015. Retrieved from <http://www.oecd.org/education/education-at-a-glance-2015.htm>
10. OECD (2018). Greece: Student performance (PISA 2018). Retrieved from <https://gpseducation.oecd.org/CountryProfile?pprimaryCountry=GRC&treshold=10&topic=PI>